Quantum Mechanics in an Entirely Deterministic Universe

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It is argued that, according to the suggested interpretation of quantum mechanical probabilities, (1) the Bell inequalities are not equivalent with those inequalities derived by Pitowsky and others that indicate the Kolmogorovity of a probability model, (2) the original Bell inequalities are irrelevant to both the question of whether or not quantum mechanics is a Kolmogorovian theory as well as the problem of determinism, whereas (3) the Pitowsky-type inequalities are not violated by quantum mechanics, hence (4) quantum mechanics is a Kolmogorovian probability theory, therefore, (5) it is compatible with an entirely deterministic universe.

1. INTRODUCTION

Is our world deterministic or indeterministic? Isn't everything already written in a big (4-dimensional) book? Is there a becoming of future events, or is time merely subjective and being timeless? Is there any difference between the ontological and epistemological probabilities? What about free will? How many universes do we want? Do we need branching histories to describe our world? All these intriguing questions of philosophy of science are related to the very basic features of quantum mechanical probabilities. The debates about determinism/indeterminism center around the problem of whether or not there exist ontological modalities. Ontological modality means that "at a given moment in the history of the world there are a variety of ways in which affairs might carry on. Before the toss of the coin there are two things that could happen, either Heads up or Tails up. This possibility is not merely epistemic, but *in re*" (Belnap and Green, n.d.). The other

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possibility is that any stochasticity is merely epistemic, related to the lack of knowledge of the states of affairs.

Many believe that physics can provide some hints for solving some of the related philosophical problems. As Reichenbach (1956) writes in connection to a very close issue, "There is no other way to solve the problem of time than ... through physics. ... If time is objective the physicist must have discovered this fact, if there is Becoming the physicist must know it; but if time is merely subjective and Being is timeless, then the physicist must have been able to ignore time in his construction of reality." However, classical (statistical) physics leaves this philosophical problem unsolved, since the stochastic models of classical statistical physics are compatible with both the assumption of an underlying ontologically deterministic as well as indeterministic theory, though, according to the common belief quantum mechanics is *not* compatible with a deterministic universe. Determinism, as it is used here, does not (necessarily) mean a functional relationship between earlier and later events, in the sense of computability, as determinism is usually understood in science (Grünbaum, 1963, pp. 314–329; Earman, 1986; Faye, 1989, Chapter 3). In other words, we do not assume that earlier time slices of the universe are determinately related, *by laws of nature*, to later time slices. Instead, determinism is used in a very deep, ontological sense. In Prior's branching time or Belnap's branching space-time terminology, there is no branching point and there is only one history (Belnap, 1992). The assumption of ontological determinism does not contradict stochastic physical theories, which reflect the emergence of indeterminism at the epistemic level. However, I want to emphasize that there is a testable physical consequence of ontological determinism. Namely, the probabilities of future events in a deterministic world should be interpreted as weighted averages of the possible truth value assignments. But we also know that this is true only if the corresponding probabilities admit a Kolmogorovian representation (Pitowsky, 1989). According to widespread opinion, quantum mechanics is not compatible even with the assumption of ontological determinism. For instance, Bub (1994) writes: "we know that we can't interpret the probabilities defined by the quantum state epistemically as measures over the different possible truth value assignments to all the propositions." This statement is, of course, true within the framework of the traditional Hilbert space quantum theory. The question is whether or not we have empirical evidence proving the nonexistence of such epistemic interpretation. As is known, the violation of Bell-type inequalities in various experiments provides such empirical evidence.

In this paper, I challenge this conclusion by showing that no rigorous proof exists that the probabilities defined by quantum mechanics should be non-Kolmogorovian. Consequently, nothing implies that quantum theory is incompatible with a deterministic universe.

2. PRELIMINARIES

2.1. Bell

An important historical step was Bell's analysis of the EPR experiment. The great advantage of Bell's approach to the problem of hidden variables was that even though he used part of the machinery of quantum mechanics, one does not need to use it, but only elementary probability calculus and the experimental results. That is why his proof of the nonexistence of (local) hidden variables has been regarded as the most serious.

Consider an experiment corresponding to the Clauser and Horn derivation of the inequalities. It is like Aspect's experiment with spin-1/2 particles (Fig. 1). Briefly recall the usual assumptions describing the intuitive notion of a local hidden variable. Assume that there is a parameter λ taken as an element of a probability space $\langle \Lambda, \Sigma(\Lambda), \rho \rangle$, such that the quantum mechanical probabilities can be represented as follows:

$$p(\lambda, A \wedge B)_{a,b} = p(\lambda, A)_a p(\lambda, B)_b$$

$$p(A)_a = \int_\Lambda p(\lambda, A)_a \, d\rho \tag{1}$$

$$p(B)_b = \int_\Lambda p(\lambda, B)_b \, d\rho$$

$$p(A \wedge B)_{a,b} = \int_\Lambda p(\lambda, A)_a p(\lambda, B)_b \, d\rho$$

For real numbers such that

$$0 \leq x, x', y, y' \leq 1$$

the following elementary inequality holds:

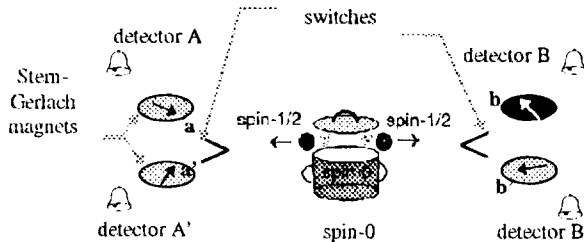$$-1 \leq xy - xy' + x'y + x'y' - x' - y \leq 0$$

Fig. 1. The Aspect experiment with spin-1/2 particles.

Applying this inequality, we have

$$-1 \leq p(\lambda, A \wedge B)_{a,b} - p(\lambda, A \wedge B')_{a,b'} + p(\lambda, A' \wedge B)_{a',b}$$
$$+ p(\lambda, A' \wedge B')_{a',b'} - p(\lambda, A')_{a'} - p(\lambda, B)_{b} \leq 0$$

Integrating this inequality, we have

$$-1 \leq p(A \wedge B)_{a,b} - p(A \wedge B')_{a,b'} + p(A' \wedge B)_{a',b} + p(A' \wedge B')_{a',b'}$$
$$- p(A')_{a'} - p(B)_{b} \leq 0 \qquad (2)$$

This is one of the well-known Clauser–Horne inequalities (one can get all the others by varying the roles of A, A', B, B'). Returning to the Aspect experiment, consider the following events:

A Left electron has spin "up" in direction **a**
A' Left electron has spin "up" in direction **a**'
B Right electron has spin "up" in direction **b**
B' Right electron has spin "up" in direction **b**'

As is well known, in the case $\angle(\mathbf{x}, \mathbf{y}) = \angle(\mathbf{y}, \mathbf{w}) = \angle(\mathbf{x}, \mathbf{w}) = 120°$ and $\mathbf{x} = \mathbf{z}$, the quantum mechanical predictions as well as the experimental results are

$$p(A)_{a} = p(A')_{a'} = p(B)_{b} = p(B')_{b'} = \tfrac{1}{2}$$
$$p(A \wedge B)_{a,b} = p(A \wedge B')_{a,b'} = p(A' \wedge B')_{a',b'} = \tfrac{3}{8} \qquad (3)$$
$$p(A' \wedge B)_{a',b} = 0$$

These probabilities violate the Clauser–Horne inequality. Thus, according to the usual conclusion, there is no local hidden variable theory reproducing the quantum mechanical probabilities.

2.2. Pitowsky

In the above derivation of Clauser–Horne inequalities, as in any other derivation of Bell-type inequalities, "the physical aspects of the problem are intermingled with the purely mathematical character of the derivation of the inequalities," says Pitowsky (1989, p. 49). He continues: "This is a course of prevailing confusion, as if Bell-type inequalities have, in themselves something to do with physics. But they do not. . . . these inequalities follow directly from the theory of probability or, if you like, from propositional logic. It is only their violation by quantum frequencies which makes them important for the foundations of physics." In the rest of this paper I challenge this conclusion at two different points. (1) I will show that Pitowsky's inequalities, derived from the Kolmogorovian probability theory, are not the same as the Bell inequalities. (2) Pitowsky's inequalities are not violated by quantum

mechanics. But, first, recall Pitowsky's important theorem about the conditions under which a probability theory is Kolmogorovian.

Let S be a set of pairs of integers $S \subseteq \{\{i, j\} \mid 1 \le i \le j \le n\}$. Denote by $R(n, S)$ the linear space of real vectors having a form like $(f_1 f_2 \cdots f_n \cdots f_{ij} \cdots)$. For each $\epsilon \in \{0, 1\}^n$, let u^ϵ be the following vector in $R(n, S)$:

$$u_i^\epsilon = \epsilon_i, \qquad 1 \le i \le n$$

$$u_{ij}^\epsilon = \epsilon_i \epsilon_j, \qquad \{i, j\} \in S \qquad (4)$$

The classical correlation polytope $C(n, S)$ (see Fig. 2) is the closed convex hull in $R(n, S)$ of vectors $\{u^\epsilon\}_{z \in \{0,1\}^n}$:

$$C(n, S) := \left\{ a \in R(n, S) \mid a = \sum_{z \in \{0,1\}^n} \lambda_z u^z \text{ such that } \lambda_z \ge 0 \text{ and } \sum \lambda_z = 1 \right\}$$

$$(5)$$

Let $\mathbf{p} = (p_1, \ldots p_n, \ldots p_{ij}, \ldots) \in R(n, S)$. We will then say that $\mathbf{p}$ *has a Kolmogorovian representation* if there exist a Kolmogorovian probability space (Ω, Σ, μ) and events $A_1, A_2, \ldots A_n \in \Sigma$ such that

$$p_i = \mu(A_i), \qquad 1 \le i \le n$$

$$p_{ij} = \mu(A_i \cap A_j), \qquad \{i, j\} \in S$$

Theorem 1 (Pitowsky, 1989). A correlation vector $\mathbf{p} = (p_1 p_2 \cdots p_n \cdots p_{ij} \cdots)$ has a Kolmogorovian representation if and only if $\mathbf{p} \in C(s, S)$.

From the definition of the polytope, equations (4) and (5), it is obvious that the condition $\mathbf{p} \in C(n, S)$ has the following meaning: the probabilities can be represented as weighted averages of the classical truth values.

In the case $n = 4$ and $S = S_4 = \{\{1, 3\}, \{1, 4\}, \{2, 3\}, \{2, 4\}\}$, the condition $\mathbf{p} \in C(n, S)$ is equivalent to the following inequalities:

$$0 \le p(A_i \wedge A_j) \le p(A_i) \le 1$$

$$0 \le p(A_i \wedge A_j) \le p(A_j) \le 1, \qquad i = 1, 2 \quad j = 3, 4$$

$$p(A_i) + p(A_j) - p(A_i \wedge A_j) \le 1$$

Fig. 2. Classical correlation polytope for $n = 2$.
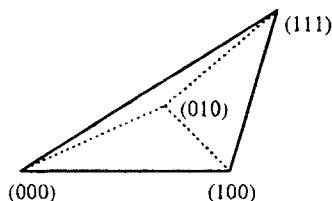
$$-1 \le p(A_1 \wedge A_3) + p(A_1 \wedge A_4) + p(A_2 \wedge A_4) - p(A_2 \wedge A_3)$$
$$- p(A_1) - p(A_4) \le 0$$
$$-1 \le p(A_2 \wedge A_3) + p(A_2 \wedge A_4) + p(A_1 \wedge A_4)$$
$$- p(A_1 \wedge A_3) - p(A_2) - p(A_4) \le 0 \tag{6}$$
$$-1 \le p(A_1 \wedge A_4) + p(A_1 \wedge A_3) + p(A_2 \wedge A_3) - p(A_2 \wedge A_4)$$
$$- p(A_1) - p(A_3) \le 0$$
$$-1 \le p(A_2 \wedge A_4) + p(A_2 \wedge A_3) + p(A_1 \wedge A_3)$$
$$- p(A_1 \wedge A_4) - p(A_2) - p(A_3) \le 0$$

The last inequality of (6) really reminds one of the inequality (2) if $A_1 = A$, $A_2 = A'$, $A_3 = B$, and $A_4 = B'$. This explains why Pitowsky calls inequalities (6) "the Clauser–Horne inequalities" and the corresponding correlation polytope $C(4, S)$ "the Clauser-Horne polytope." Therefore, substituting for the probabilities in the last inequality of (6) the same values as were calculated from quantum mechanics in (3), we have

$$\frac{3}{8} + \frac{3}{8} + \frac{3}{8} - 0 - \frac{1}{2} - \frac{1}{2} = \frac{1}{8} > 0 \tag{7}$$

Consequently,

$$\mathbf{p} = \left(\frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{3}{8} \frac{3}{8} 0 \frac{3}{8} \right) \notin C(4, S)$$

Now, Pitowsky concludes: "We have demonstrated that $\mathbf{p} \notin C(n, S)$ and therefore we cannot explain the statistical outcome by assuming that the source is an "urn," containing electron pairs in the singlet state, such that the distribution of the properties A, A', B, B' in this "urn" is fixed before the measurement" (Pitowsky, 1989, p. 82; notation changed for sake of uniformity). "The violations of these constraints on correlations by quantum frequencies thus poses a major problem for all schools of classical probability. I take this fact to be the major source of difficulty which underlies the interpretation of quantum theory" (Pitowsky, 1989, p. 87).

3. THE NONEQUALITY OF THE BELL INEQUALITIES

At this essential point I challenge the above conclusion of Pitowsky: in formula (7) we substituted *incorrect values* for the probabilities of inequality (6). In order to see why, we have to investigate the exact meaning of the probabilities in quantum mechanics and in Pitowsky's inequalities.

Whenever we talk about probabilities *we have to specify the system of conditions under which the probabilities are understood.* Sometimes these conditions are given only tacitly, but without such conditions the probabilities are meaningless. The meaning of the quantum mechanical probability, $tr(WA)$, is this: "the probability of getting outcome A *given* that the measurement a is performed." Let $p(A)_a$ denote such a conditional probability. Throughout the derivation of the original Clauser–Horne inequalities (2) we used conditional probabilities. Making the tacit conditions explicit, (Fig. 3.), we have

$$-1 \le p(A \wedge B)_{a,b} - p(A \wedge B')_{a,b'} + p(A' \wedge B)_{a',b}$$
$$+ p(A' \wedge B')_{a',b'} - p(A')_{a'} - p(B)_b \le 0 \qquad (9)$$

It is remarkable that in the original Clauser–Horne inequality (9) we have probabilities corresponding to *different* conditions.

At the same time the probabilities in the Pitowsky theorem belong to *one common set of conditions*, as a consequence of the fact that the probability measure on any Kolmogorovian probability space also belongs to one common set of conditions. These common conditions can be formulated in various ways. In the case of the EPR experiment they can be the prepared physical circumstances at the moment of decay of the source particle. For brevity this primary common set of conditions is not indicated in our future notations. Since A entails a, one can, of course, calculate the "unconditioned" probabilities in the usual way:

$$p(A) = tr(WA) \cdot p(a - \text{measurement}) \qquad (10)$$

Actually these are the probabilities that one can measure in a laboratory by counting the "beeps" given by a detector responsible for outcome A. Assume

$$-1 \le p(A \wedge B)_{a,b} - p(A \wedge B')_{a,b'} + p(A' \wedge B)_{a',b} + p(A' \wedge B')_{a',b'}$$
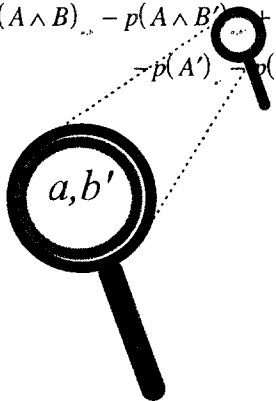$$- p(A')_{a'} - p(B)_b \le 0 \qquad (2)$$

Fig. 3. The smaller than usual subscripts through Section 2.1. symbolize that those conditions are always there, but neglected by people.

that the switches (Fig. 1) choose between the corresponding two measurements with equal probability: $p(a) = p(a') = p(b) = p(b') = 1/2$. So, the correct numbers in (7) should be

$$\frac{3}{32} + \frac{3}{32} + \frac{3}{32} - 0 - \frac{1}{4} - \frac{1}{4} = -\frac{7}{32} \in [-1, 0] \tag{11}$$

Consequently,

$$\mathbf{p} = \left(\frac{1}{4}\ \frac{1}{4}\ \frac{1}{4}\ \frac{1}{4}\ \frac{3}{32}\ \frac{3}{32}\ 0\ \frac{3}{32} \right) \in C(4, S_4) \tag{12}$$

This means that the *measured probabilities* in the EPR experiment *are Kolmogorovian*!

Thus, the original Bell (Clauser–Horne) inequalities, such as (9), which contain conditional probabilities that belong to different conditions, *are not the same inequalities* as Pitowsky's "Bell (Clauser–Horne)" inequalities like (6).

4. IS QUANTUM MECHANICS COMPATIBLE WITH A DETERMINISTIC UNIVERSE?

The fact that Pitowsky's inequalities are not violated leads to the conclusion that *the EPR experiments do not provide empirical evidence against the Kolmogorovian character of quantum probabilities.* This conclusion is entirely contrary to that of Pitowsky himself.[3] One can ask now several questions: (1) What is the conceptual origin of Pitowsky's argumentation? (2) What is the deeper meaning of the fact that the EPR experiments do not violate Pitowsky's version of "Clauser–Horne" inequalities, while they do violate the original Clauser–Horne inequalities? (3) If, as I suggest, quantum mechanics is a Kolmogorovian probability theory, what kind of consequences follow from this fact with respect to the hidden variable theories and the possibility of a deterministic explanation of quantum phenomenon?

4.1. Presumptions Versus Reality

In order to see the conceptual mistake in substituting the value of conditional probability $p(A)_a$ for the "unconditioned" probability $p(A)$, consider two distinct understandings of quantum mechanical probabilities:

(a) $p(A)_a = tr(WA)$ is the probability of the occurrence of the outcome A given that the measurement a is performed.

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[3] It also contradicts some parts of my two earlier papers (Szabó, 1993, 1994) in which I uncritically recalled the conclusions of Pitowsky.

(b) $p(\tilde{A}) = \mathbf{tr}(WA)$ is the probability that the system has the *property* $\tilde{A}$, which property consists in that "the outcome A occurs whenever measurement a is performed."

If we accept version (a), then the substitution $p(A)_a$ for $p(A)$ is a simple mistake of calculation. To assume (b) leads to the following contradiction. In the case of the EPR experiment:

(i) According to interpretation (b), one assumes that: "the source is an 'urn,' containing electron pairs in singleton state such that the distribution of the properties A, A', B, B' in this 'urn' is fixed before the measurement."

(ii) In this case one can correctly substitute $p(A)_a$ for $p(A)$, and have

$$\mathbf{p} \notin C(n, S)$$

(iii) From which it follows that: "we cannot explain the statistical outcome by assuming that the source is an 'urn,' containing electron pairs in singleton state such that the distribution of the properties A, A', B, B' in this 'urn' is fixed before the measurement."

From the contradiction between the quoted statements in (i) and (iii) it does not at all follow that quantum mechanics is a non-Kolmogorovian probability theory. But it does follow that (b) is an untenable interpretation of the quantum mechanical probabilities.

4.2. Violation of the Original Inequalities

One can ask, then, what does the violation of the original Bell-type inequalities mean, if does not mean that quantum mechanics is a non-Kolmogorovian theory? Compare the two kinds of Bell inequalities. Taking into account (10), the original Clauser–Horne inequalities can be expressed in the following way:

$$-1 \leq \frac{p(A \wedge B)}{p(a \wedge b)} - \frac{p(A \wedge B')}{p(a \wedge b')} + \frac{p(A' \wedge B)}{p(a' \wedge b)} + \frac{p(A' \wedge B')}{p(a' \wedge b')}$$

$$- \frac{p(A')}{p(a')} - \frac{p(B)}{p(b)} \leq 0 \tag{13}$$

The corresponding Pitowsky inequality is the last inequality of (6):

$$-1 \leq p(A \wedge B) - p(A \wedge B') + p(A' \wedge B) + p(A' \wedge B')$$

$$- p(A') - p(B) \leq 0 \tag{14}$$

However, inequalities (13) and (14) are entirely different. And there is no easy way to specify when the violation of one of them involves the violation of the other. Fortunately we do not need to deal with this problem since we

can entirely neglect the original Bell inequalities. We can do that for two different reasons. First, as we have seen, they are completely irrelevant to the question of whether or not quantum mechanics is a Kolmogorovian theory. Second, they are irrelevant from the point of view of a common cause explanation of quantum correlations. Indeed, according to the understanding of quantum mechanical probabilities mentioned as version (a) in Section 4.1, the question of whether or not there exists such a hidden parameter that satisfies conditions

$$p(\lambda, A \wedge B)_{a,b} = p(\lambda, A)_a p(\lambda, B)_b$$

$$p(A)_a = \int_\Lambda p(\lambda, A)_a \, d\rho$$

$$p(B)_b = \int_\Lambda p(\lambda, B)_b \, d\rho$$

$$p(A \wedge B)_{a,b} = \int_\Lambda p(\lambda, A)_a p(\lambda, B)_b \, d\rho \tag{15}$$

is meaningless. As a matter of fact, these conditions would express the intuitive conception of a common cause mechanism responsible for the EPR correlations only if the probabilities in (15) were interpreted as the probabilities of appearances of "properties" of the system. This interpretation is nothing else but the version (b) in Section 4.1. Only in this case could we infer from the violation of the original Clauser–Horne inequality to the violation of Einstein locality. But we have rejected version (b) as a contradictory interpretation. In other words, since there are no "properties" corresponding to outcomes of measurements we can perform on the system (at least quantum mechanics has nothing to do with such properties), we do not need to explain "the correlation between spatially separated occurrence of such [nonexisting] properties." But, there do exist (observable physical) events corresponding to performance-preparations of various measurements and other events which correspond to the outcomes. Each such event occurs with certain probability. What we observe in the EPR experiment is a correlation between spatially separated *outcomes*. And the question is whether or not a local hidden variable explanation is possible for such a correlation. Thus, the whole formulation of the common cause problem has to be reconsidered.

4.3. Correct Formulation of the Local Hidden Variable Problem

A local hidden variable theory (considered as a mathematically well formulated representation of a deterministic and nonviolating-Einstein-causality universe) has to reproduce the probabilities of the outcomes and

that of the performance-preparation. The assumed "parameter" $\lambda \in \Lambda$ should represent the state of the part of the universe which belongs to the common past of the two separated measurements (Fig. 4) such that for both the outcomes and the performance-preparations we have

$$p(A) = \int_\Lambda p(\lambda, A) \, d\rho \qquad (16/1)$$

$$p(A') = \int_\Lambda p(\lambda, A') \, d\rho \qquad (16/2)$$

$$p(B) = \int_\Lambda p(\lambda, B) \, d\rho \qquad (16/3)$$

$$p(B') = \int_\Lambda p(\lambda, B') \, d\rho \qquad (16/4)$$

$$p(a) = \int_\Lambda p(\lambda, a) \, d\rho \qquad (16/5)$$

$$p(a') = \int_\Lambda p(\lambda, a') \, d\rho \qquad (16/6)$$

$$p(b) = \int_\Lambda p(\lambda, b) \, d\rho \qquad (16/7)$$

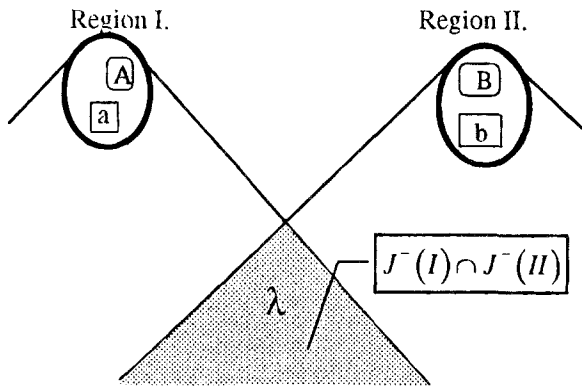$$p(b') = \int_\Lambda p(\lambda, b') \, d\rho \qquad (16/8)$$

Fig. 4. The parameter $\lambda \in \Lambda$ represents the state of the universe in the common past of regions I and II.

The same holds for the conjunctions

$$p(A \wedge B) = \int_\Lambda p(\lambda, A \wedge B) \, d\rho$$

$$p(A \wedge B') = \int_\Lambda p(\lambda, A \wedge B') \, d\rho \tag{16/9}$$

$$\vdots \tag{16/10}$$

$$p(a' \wedge B') = \int_\Lambda p(\lambda, a' \wedge B') \, d\rho$$

We also assume that the underlying hidden variable theory is Einstein-local. This means that the correlation between any two spatially separated events should be the consequence of the λ dependence of the corresponding probabilities and not the consequence of a direct physical interaction. We can formulate this assumption via the following relations:

$$p(\lambda, A \wedge B) = p(\lambda, A) \cdot p(\lambda, B) \tag{17/1}$$

$$p(\lambda, A \wedge B') = p(\lambda, A) \cdot p(\lambda, B') \tag{17/2}$$

$$p(\lambda, A' \wedge B) = p(\lambda, A') \cdot p(\lambda, B) \tag{17/3}$$

$$p(\lambda, A' \wedge B') = p(\lambda, A') \cdot p(\lambda, B') \tag{17/4}$$

$$p(\lambda, a \wedge b) = p(\lambda, a) \cdot p(\lambda, b) \tag{17/5}$$

$$p(\lambda, a \wedge b') = p(\lambda, a) \cdot p(\lambda, b') \tag{17/6}$$

$$p(\lambda, a' \wedge b) = p(\lambda, a') \cdot p(\lambda, b) \tag{17/7}$$

$$p(\lambda, a' \wedge b') = p(\lambda, a') \cdot p(\lambda, b') \tag{17/8}$$

$$p(\lambda, A \wedge b) = p(\lambda, A) \cdot p(\lambda, b) \tag{17/9}$$

$$p(\lambda, A \wedge b') = p(\lambda, A) \cdot p(\lambda, b') \tag{17/10}$$

$$p(\lambda, A' \wedge b) = p(\lambda, A') \cdot p(\lambda, b) \tag{17/11}$$

$$p(\lambda, A' \wedge b') = p(\lambda, A') \cdot p(\lambda, b') \tag{17/12}$$

$$p(\lambda, a \wedge B) = p(\lambda, a) \cdot p(\lambda, B) \tag{17/13}$$

$$p(\lambda, a \wedge B') = p(\lambda, a) \cdot p(\lambda, B') \tag{17/14}$$

$$p(\lambda, a' \wedge B) = p(\lambda, a') \cdot p(\lambda, B) \tag{17/15}$$

$$p(\lambda, a' \wedge B') = p(\lambda, a') \cdot p(\lambda, B') \tag{17/16}$$

Relations (17/1)–(17/4) express the λ-level independence of the outcomes (in other words, the screening off the outcomes by the hidden parameter), relations (17/5–17/8) express the λ-level independence of the choices of which measurement will be performed. Finally, equations (17/9)–(17/16) represent the λ-level independence of the outcomes from the spatially separated choices (required parameter independence).

Now, the question is whether there exists such a parameter satisfying conditions (16/1–16/24) and (17/1)–(17/16). One can apply the following[4]:

Theorem 2. With the notation of the Section 2.2, consider events A_1, $\ldots$, A_n and a set of indexes S. Assume that a correlation vector $\mathbf{p} = (p(A_1)$ $\cdots p(A_n) \cdots p(A_i \wedge A_j) \cdots)$ can be represented as a convex combination of parameter-dependent correlation vectors $(\pi(\lambda, A_1) \cdots \pi(\lambda, A_n) \cdots \pi(\lambda, A_i \wedge A_j) \cdots)$,

$$p(A_i) = \int_\Lambda \pi(\lambda, A_i)\, d\rho(\lambda) \qquad \text{for} \quad 1 \leq i \leq n \tag{18}$$

$$p(A_i \wedge A_j) = \int_\Lambda \pi(\lambda, A_i \wedge A_j)\, d\rho(\lambda) \qquad \text{for} \quad \{i, j\} \in S$$

such that

$$\pi(\lambda, A_i \wedge A_j) = \pi(\lambda, A_i) \cdot \pi(\lambda, A_j) \qquad \text{for each} \quad \{i, j\} \in S \tag{19}$$

Then $\mathbf{p} \in C(n, S)$.

See Szabó (1995) for the proof.

Equations (16/1)–(16/24) correspond to (18) and (17/1)–(17/16) to (19) in the case

$$A_1 = A \qquad A_5 = a$$
$$A_2 = A' \qquad A_6 = a'$$
$$A_3 = B \qquad A_7 = b$$
$$A_4 = B' \qquad A_8 = b'$$

$S = S_{16} = \{\{1,3\}, \{1,4\}, \{2,3\}, \{2,4\}, \{5,7\}, \{5,8\}, \{6,7\},$
$\qquad\qquad \{6,8\}, \{1,7\}, \{1,8\}, \{2,7\}, \{2,8\}, \{5,3\}, \{5,4\}, \{6,3\}, \{6,4\}\}$

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[4] We cannot use the formal reproduction of the Clauser–Horne derivation described in Section 2.1 because the number of events and the number of investigated conjunctions are larger than four.

According to this theorem, if there exists a hidden parameter theory satisfying conditions (16/1)–(16/24) and (17/1)–(17/16), then the *observed* probabilities should satisfy condition $\mathbf{p} \in C(8, S_{16})$.

Let us gather all the information we know about the observed probabilities in the Aspect-type spin-correlation experiment. According to the Section 3, we have

$$p(A) = p(A') = p(B) = p(B') = \frac{1}{4} \tag{20/1}$$

$$p(a) = p(a') = p(b) = p(b') = \frac{1}{2} \tag{20/2}$$

$$p(A \wedge B) = p(A \wedge B') = p(A' \wedge B') = \frac{3}{32} \tag{20/3}$$

$$p(A' \wedge B) = 0 \tag{20/4}$$

The probabilities with which the different measurements are chosen satisfy the following independence conditions:

$$p(a \wedge b) = p(a) \cdot p(b)$$

$$p(a \wedge b') = p(a) \cdot p(b')$$

$$p(a' \wedge b) = p(a') \cdot p(b)$$

$$p(a' \wedge b') = p(a') \cdot p(b')$$

Therefore

$$p(a \wedge b) = p(a \wedge b') = p(a' \wedge b) = p(a' \wedge b') = \frac{1}{4} \tag{20/5}$$

Similarly, it is an observed fact that the outcomes on the one side are independent of the choices on the other side (observed parameter independence). This means that

$$p(A \wedge b) = p(A) \cdot p(b) \qquad\qquad p(a \wedge B) = p(a) \cdot p(B)$$

$$p(A \wedge b') = p(A) \cdot p(b') \qquad\qquad p(a \wedge B') = p(a) \cdot p(B')$$

$$p(A' \wedge b) = p(A') \cdot p(b) \qquad\qquad p(a' \wedge B) = p(a') \cdot p(B)$$

$$p(A' \wedge b') = p(A') \cdot p(b') \qquad\qquad p(a' \wedge B') = p(a') \cdot p(B')$$

According to these independence relations,

$$p(A \wedge b) = p(A \wedge b') = p(A' \wedge b) = p(A' \wedge b')$$

$$= p(a \wedge B) = p(a \wedge B') = p(a' \wedge B) = p(a' \wedge B') = \frac{1}{8} \tag{20/6}$$

We can collect the data from (20/1)–(20/6) in a correlation vector. The question is whether or not this correlation vector is contained in the classical correlation polytope:

$$\mathbf{p} = (\tfrac{1}{4}\,\tfrac{1}{4}\,\tfrac{1}{4}\,\tfrac{1}{4}\,\tfrac{1}{2}\,\tfrac{1}{2}\,\tfrac{1}{2}\,\tfrac{1}{2}\,\tfrac{3}{32}\,\tfrac{3}{32}\; 0\; \tfrac{3}{32}\,\tfrac{1}{4}\,\tfrac{1}{4}\,\tfrac{1}{4}\,\tfrac{1}{8}\,\tfrac{1}{8}\,\tfrac{1}{8}\,\tfrac{1}{8}\,\tfrac{1}{8}\,\tfrac{1}{8}\,\tfrac{1}{8}) \overset{?}{\in} C(8, S_{16}) \qquad (21)$$

In the case $n > 4$ there are no derived inequalities which would be equivalent to the condition $\mathbf{p} \in C(n, S)$ [see Pitowsky (1989) for the details]. We thus have to examine the geometric condition (21) directly. I tested condition (21) by computer and the result is affirmative:

$$\mathbf{p} = (\tfrac{1}{4}\,\tfrac{1}{4}\,\tfrac{1}{4}\,\tfrac{1}{4}\,\tfrac{1}{2}\,\tfrac{1}{2}\,\tfrac{1}{2}\,\tfrac{1}{2}\,\tfrac{3}{32}\,\tfrac{3}{32}\; 0\; \tfrac{3}{32}\,\tfrac{1}{4}\,\tfrac{1}{4}\,\tfrac{1}{4}\,\tfrac{1}{8}\,\tfrac{1}{8}\,\tfrac{1}{8}\,\tfrac{1}{8}\,\tfrac{1}{8}\,\tfrac{1}{8}\,\tfrac{1}{8}) \in C(8, S_{16})$$

Consequently, *there is no proved disagreement between the assumptions* (16/1)–(16/24) *and* (17/1)–(17/16) *about a local hidden variable theory and the observations.* In other words, *the existence of a local hidden variable theory is not excluded.*

5. CONCLUSIONS

From my historical review it turned out that the Bell analysis of the EPR experiment was considered to be a clear and indeed the best illustration of the alleged fact that there is no (at least local) hidden variable theory which can reproduce the quantum mechanical probabilities. We recalled Pitowsky's theorem about the conditions under which a probability theory is Kolmogorovian. It is widely believed that Pitowsky's conditions are, as he claimed himself, equivalent with the Bell inequalities. Since quantum probabilities violate these inequalities, it is believed that quantum mechanics is not only incompatible with a local hidden parameter theory, but also incompatible with a Kolmogorovian probability-theoretic description. As a consequence of the non-Kolmogorovity, it is also maintained that quantum theory is incompatible with an ontologically deterministic world, i.e., the probabilities defined by the quantum states cannot be interpreted epistemically as weighted averages of the different possible truth value assignments to all the propositions. However, we have shown the following:

1. The Bell inequalities are not equivalent with the inequalities derived by Pitowsky.
2. The original Bell inequalities are irrelevant.
3. The Pitowsky-type inequalities are not violated by quantum mechanics.
4. Quantum mechanics is a Kolmogorovian probability theory.
5. A more correct formulation of the hidden variable problem can be given.

6. There is no rigorous indication that quantum mechanics is not compatible with an entirely deterministic universe.

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REFERENCES

Bell, J. S. (1987). *Speakable and Unspeakable in Quantum Mechanics. Collected Papers on Quantum Philosophy*, Cambridge University Press, Cambridge.

Belnap, N. (1992). Branching space-time, *Synthese*, **92**, 385–434.

Belnap, N., and Green, M. (n.d.). Indeterminism and the thin red line, in *Philosophical Perspectives*, James E. Tomberlin, ed., pp. 7–8.

Bub, J. (1994). On the structure of quantum proposition systems, *Foundations of Physics*, to appear.

Earman, J. (1986). *A Primer on Determinism*, Reidel, Dordrecht.

Faye, J. (1989). *The Reality of the Future*, Odense University Press, Odense.

Grünbaum, A. (1963). *Philosophical Problems of Space and Time*, Knopf, New York.

Maxwell, N. (1985). Are probabilism and special relativity incompatible? *Philosophy of Science*, **52**, 23–43.

Pitowsky, I. (1989). *Quantum Probability–Quantum Logic*, Springer, Berlin.

Reichenbach, H. (1956). *The Direction of Time*, University of California Press, Berkeley.

Stein, H. (1991). On relativity theory and openness of the future, *Philosophy of Science*, **58**, 147–167.

Szabó, L. E. (1993). On the real meaning of Bell's theorem, *Foundations of Physics Letters*, **6**, 191.

Szabó, L. E. (1994). On the real meaning of Bell's theorem, *International Journal of Theoretical Physics*, **33**, 191.

Szabó, L. E. (1995). Is quantum mechanics compatible with a deterministic universe? Two interpretations of quantum probabilities, *Foundations of Physics Letters*, to appear.